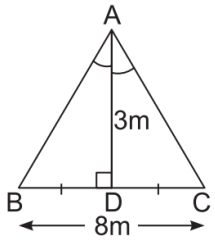
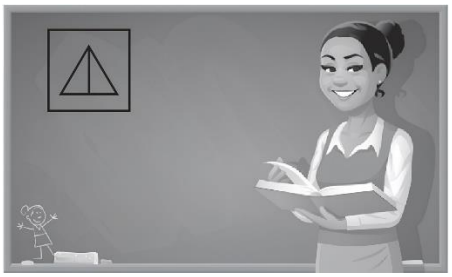
Triangles

Case Study Based Questions

Case Study 1

One day, a geometry teacher came in the class and she decided to teach students about the triangle. She was considering a Mathematic problem for students; in ΔABC, AD is the perpendicular bisector of BC.





She asked the following questions from the students. On the basis of the above information, solve the following questions.

Q1. Δ ADB is congruent to Δ ADC by:

a. AAS axiom b. ASA axiom

c. RHS axiom d. SAS axiom

Q2. Δ ABC is a/an:

a. right angled triangle

b. isosceles triangle

c. equilateral triangle

d. isosceles right angled triangle

Q3. In given figure, length of AB in Δ ABC is:

a. 6 m b. 4 m

c. 5 m d. 7 m

Q4. The area of Δ ABC is:

a. 9 m^2 b. 10 m^2

c. 12 m^2 d. 14 m^2

Q5. Which of the following axiom is not satisfy the condition of congruency?

a. RHS b. ASA

c. SSS d. SSA

Solutions

1. (d) In ΔADB and ΔADC

AD = AD [Common]

BD = DC

[∵ AD is perpendicular bisector of BC]

and ∠ADB = ∠ADC = 90°

∴ ΔADB ≅ ΔADC by SAS axiom.

So, option (d) is correct.

2. (b) ∵ ΔADB ≅ ΔADC,

∴ AB = AC [By CPCT]

Hence, ΔABC is an isosceles triangle.

So, option (b) is correct.

3. (c) In right angled ΔADB, use Pythagoras theorem

$$AB = \sqrt{(BD)^2 + (AD)^2} = \sqrt{(4)^2 + (3)^2}$$

$$\left[\because BD = DC = \frac{8}{2} = 4 \text{ m}\right]$$

[Common side]

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ m}$$

So, option (c) is correct.

4. (c) $\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD$

$$= \frac{1}{2} \times 8 \times 3 = 12 \text{ m}^2$$

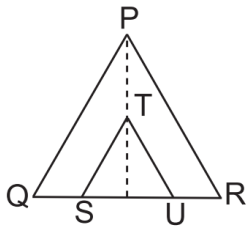
So, option (c) is correct.

5. (d) SSA axiom is not the condition of congruency.

Case Study 2

Red Pyramid (or North Pyramid) is the largest pyramid in cairo Egypt's capital. It is a rusty red pyramid made up of red limestones. The height of the pyramid is 105 m and its base is 220 m.

It is also believed that it was Egyptians first successfuly attempt at constructing a smooth side pyramid. It can be seen that two triangles ΔPQR and ΔSTU have been marked in the images of pyramid, such that ST ∥ PQ and TU ∥ PR and ∠P = ∠T.



On the basis of the above information, solve the following questions.

Q 1. Which of the following is not a criteria for congruency?

a. SSS b. SSA c. ASA d. RHS

Q 2. If ∠TSU = 40° and TS = TU, then ∠STU is:

a. 100° b. 110°
c. 120° d. 130°

Q 3. If ∠QPR = 50° and QP = PR, then ∠PQR is:

a. 65° b. 70°
c. 75° d. 80°

Q 4. If hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle then the two triangles are congruent by axiom.

a. SSS b. RHS

c. SAS d. ASA

Q 5. Area of two congruent figures is:

a. equal b. not equal

c. (a) or (b) d. do not say anything

Solutions

1. (b) SSA is not a criteria for congruency.
So, option (b) is correct.

2. (a) In ΔTSU,

$$TS = TU \quad \text{(Given)}$$

$$\Rightarrow \quad \angle U = \angle S = 40°$$

(Angles opposite to equal sides are equal)

$$\therefore \quad \angle T + \angle S + \angle U = 180°$$

$$\angle T + 40° + 40° = 180°$$

$$\Rightarrow \quad \angle T = 100$$

So, option (a) is correct.

3. (a) In ΔPQR,

$$QP = PR \quad \text{[Given]}$$

$$\Rightarrow \quad \angle R = \angle Q$$

$$\because \quad \angle P + \angle Q + \angle R = 180°$$

$$\therefore \quad \angle P + \angle Q + \angle Q = 180°$$

$$50° + 2\angle Q = 180°$$

$$\Rightarrow \quad 2\angle Q = 130° \quad \Rightarrow \quad \angle Q = 65°$$

So, option (a) is correct.

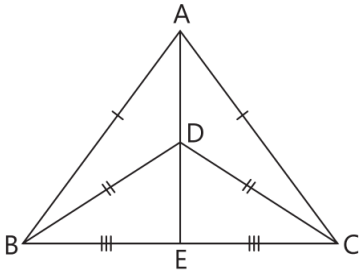
4. (b) RHS
So, option (b) is correct.

5. (a) Area of two congruent figures is always equal.
So, option (a) is correct.

Case Study 3

Sunil is a farmer who is having a triangular plot. As the land is limited with him, so he decided to divide his land in four parts so that he can use multiple cropping method in which fields are growing with different seads. The field is divided as shown below.

In this figure, ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. Line AD is extended to BC, which intersect at point E.



On the basis of the above information, solve the following questions.

Q 1. If Sunil decides to grow crop in ΔABD, then which of the other triangle is of exact shape and size?

a. $\Delta ABD \cong \Delta ACD$ b. $\Delta ABD \cong \Delta BDE$
c. $\Delta ABD \cong \Delta ACE$ d. $\Delta ACD \cong \Delta AEB$

Q 2. In ΔBDC, if $\angle B + \angle D + \angle C = 180°$, $\angle D = 70°$, then $\angle B$ is equal to:

a. 70° b. 60° c. 55° d. 80°

Q 3. If Sunil decides to grow multiple crop in ΔBDE, then which of the other triangle is of exact shape and size?

a. $\Delta BDE \cong \Delta CDE$ b. $\Delta BDE \cong \Delta CED$
c. $\Delta BED \cong \Delta CDE$ d. $\Delta CED \cong \Delta EBD$

Q 4. Which of the following pair is congruent?

a. ΔABE and ΔADC b. ΔABE and ΔACE
c. ΔAEC and ΔADB d. ΔABE and ΔABD

Q 5. Two circles with same radii but different centres, then they are:

a. not always congruent
b. always congruent
c. congruent in some special cases
d. congruent when areas are different

Solutions

1. (a) In ΔABD and ΔACD

$AB = AC$ [Given]
$BD = DC$ [Given]
$AD = AD$ [Common]
Hence, $\Delta ABD \cong \Delta ACD$. [By SSS rule]
So, option (a) is correct.

2. (c) Given $\angle D = 70°$

$\because \quad \angle B + \angle D + \angle C = 180°$
$\therefore \quad \angle B + 70° + \angle B = 180°$
$[\because BD = DC \Rightarrow \angle C = \angle B]$
$\Rightarrow \quad 2\angle B = 110°$
$\Rightarrow \quad \angle B = 55°$
So, option (c) is correct.

3. (a) In ΔBDE and ΔCDE,

$BD = CD$ [Given]
$DE = DE$ [Common]
and $BE = EC$ [Given]

$\therefore$ $\Delta BDE \cong \Delta CDE$ [By SSS rule]

So, option (a) is correct.

4. (b) In ΔABE and ΔACE,

$AB = AC$

$BE = EC$

and $AE = AE$ [Common]

$\therefore$ $\Delta ABE \cong \Delta ACE$ [By SSS rule]

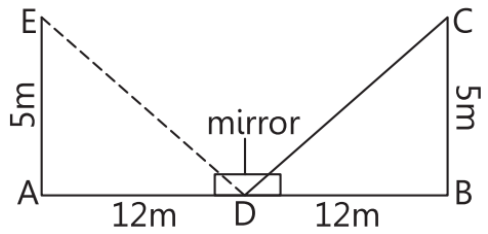
So, option (b) is correct.

5. (b) Two circles with same radii but different centres are always congruent.

So, option (b) is correct.

Case Study 4

John recently read a Mathematics experiment. He was keen to perform it on its own. He choosed a long building whose height he want to know, he placed a mirror at ground. He is standing at some distance to the building as well as mirror. John height is 5 m and the distance of John from mirror is 12 m and distance of building from mirror is also 12 m and its height 5 m.



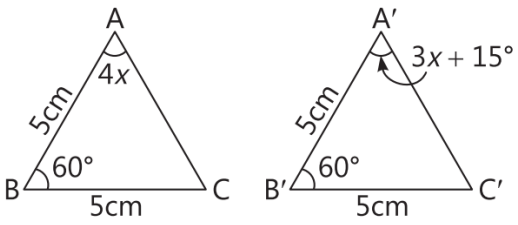
On the basis of the above information, solve the following questions.

Q 1. Write two congruent triangles formed in the given figure.

Q 2. Find the distance between top of building and mirror.

Q 3. Find the area of ΔAED.

Q 4. In the given figures, find the measure of $\angle B'A'C'$.



Solutions

1. In ΔAED and ΔBCD,

$AD = BD = 12$ m

$AE = BC = 5$ m

and $\angle A = \angle B = 90°$

$\therefore$ $\Delta AED \cong BCD$ [By SAS rule]

2. In right angled ΔEAD, use Pythagoras theorem,

$$ED = \sqrt{(AE)^2 + (AD)^2}$$

$$= \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144}$$

$$= \sqrt{169} = 13 \text{ m}.$$

Hence, the distance between top of building and mirror is 13 m.

3. $$\text{Area of } \Delta AED = \frac{1}{2} \times AD \times AE$$

$$= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$$

4. In ΔABC and $\Delta A'B'C'$

$AB = A'B' = 5$ cm [Given]

$\angle B = \angle B' = 60°$ [Given]

and $BC = B'C' = 5$ cm

$\therefore$ $\Delta ABC \cong \Delta A'B'C'$ [By SAS congruence]

$\Rightarrow$ $\angle BAC = \angle B'A'C'$ [By CPCT]

$\Rightarrow$ $4x = 3x + 15°$

$\Rightarrow$ $x = 15°$

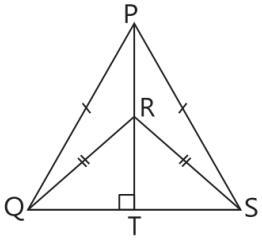
$\therefore \angle B'A'C' = 3x + 15°$

$= 3 \times 15° + 15°$ [$\because x = 15°$]

$= 45° + 15° = 60°$

Case Study 5

Kushal is a property dealer. He has been in this business from past 12 yr. He bought a triangular shaped plot. It divides the plot into four parts which is shown in the figure.



On the basis of the above information, solve the following questions.

Q 1. If Sunil decides to sell two plots, then show that two triangles ΔPQR and ΔPSR have exact shape and size?

Q 2. In ΔQRS, ∠RQS = 50°, then find ∠QSR.

Q 3. If ∠QPS = 60°, then find ∠QPT.

Solutions

1. In ΔPQR and ΔPSR,

PQ = PS [Given]

QR = RS [Given]

and PR = PR [Common]

∴ ΔPQR ≅ ΔPSR [By SSS congruency]

2. In ΔQRS, ∠RQS = 50°

Since QR = RS

⇒ ∠QSR = ∠RQS = 50°

[∵ angles opposite to equal sides are equal]

3. In ΔQPT and ΔSPT,

PQ = PS [Given]

PT = PT [Common]

and ∠QTP = ∠STP = 90°

∴ ΔQPT ≅ ΔSPT [By RHS Congruence rule]

⇒ ∠QPT = ∠SPT [By CPCT]

∵ ∠QPS = 60°

⇒ 2∠QPT = 60° ⇒ ∠QPT = 30°